MATHEMATICAL SIMULATIONS IN 3D ANIMATION

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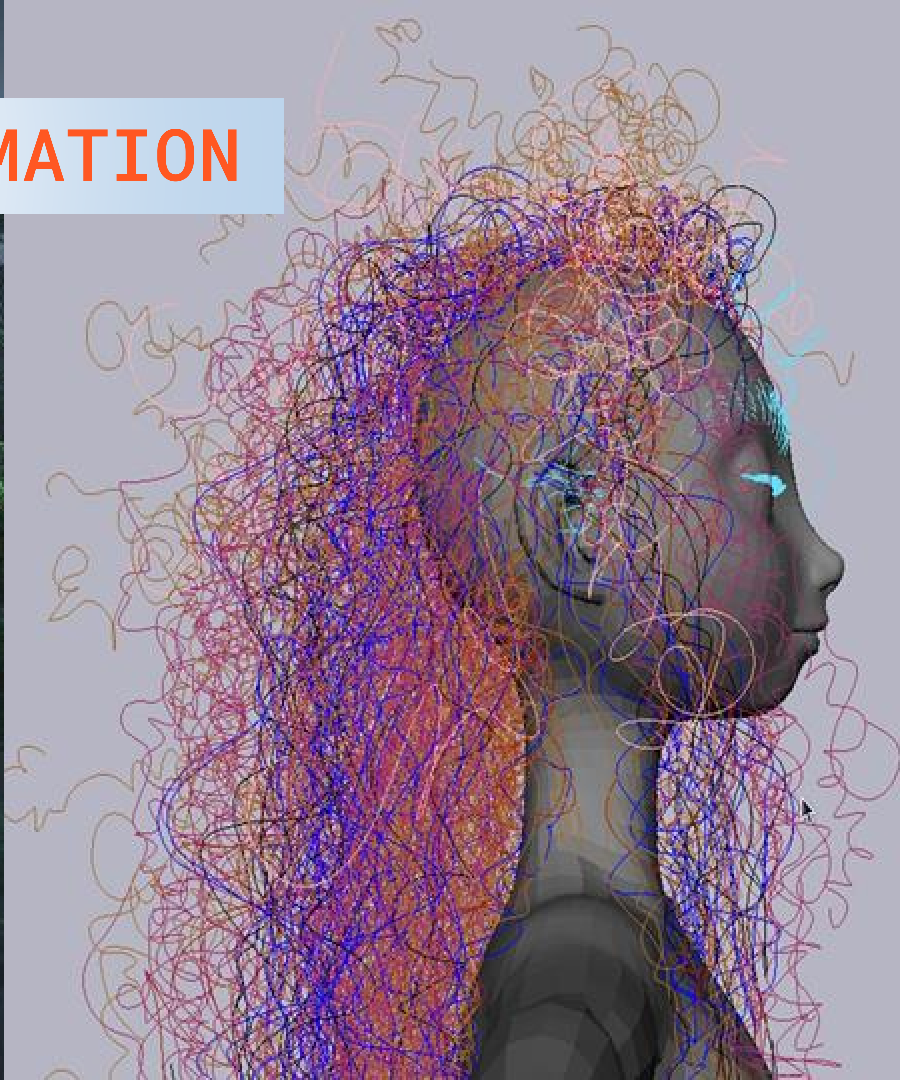
OVERVIEW

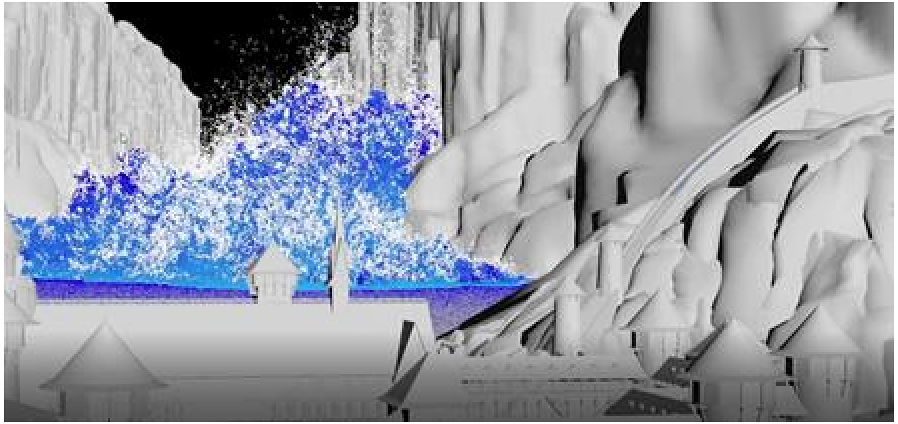
- What are simulations in animation?
- Why are they useful?
- Character simulation: Clothing and hair
- Creating a simple interactive 3D curtain

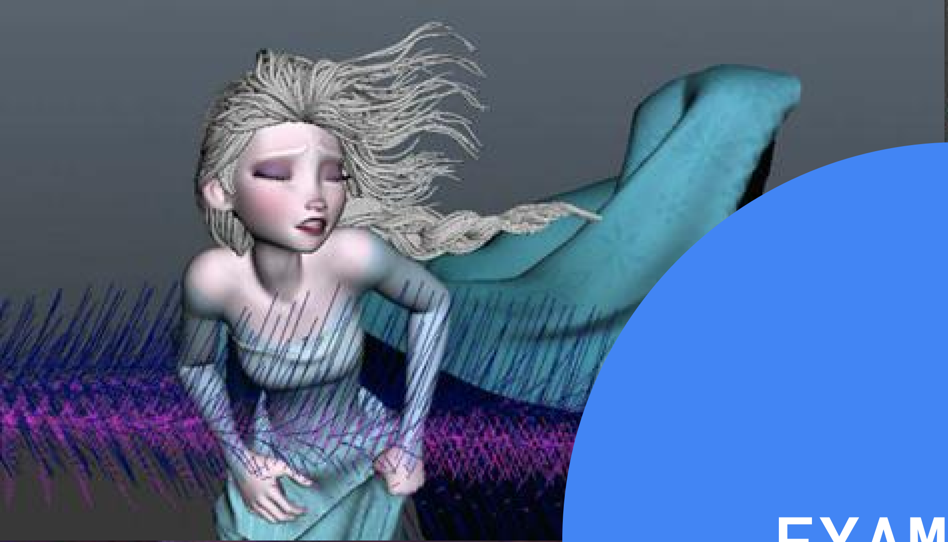
SIMULATIONS IN 3D ANIMATION

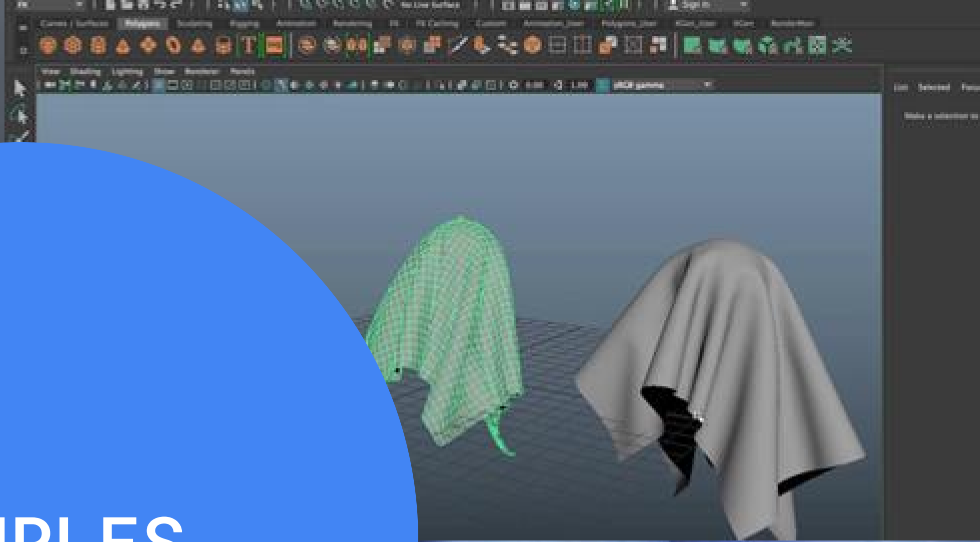


CATEGORIES OF SIMULATION

- PARTICLE SYSTEM SFX
  - WATER, SMOKE, FIRE, WIND, ETC.
- CHARACTER EFFECTS
  - CLOTHING, HAIR
- LIGHTING AND SHADING
- CROWDS + COMBINATORICS







EXAMPLES

WHAT KIND OF MATH GOES INTO SIMULATIONS?

- A LOT!
  - Math is employed in many different ways with simulations. Today I will primarily be discussing: algebra, geometry, discrete

WHERE IS THE MATH?

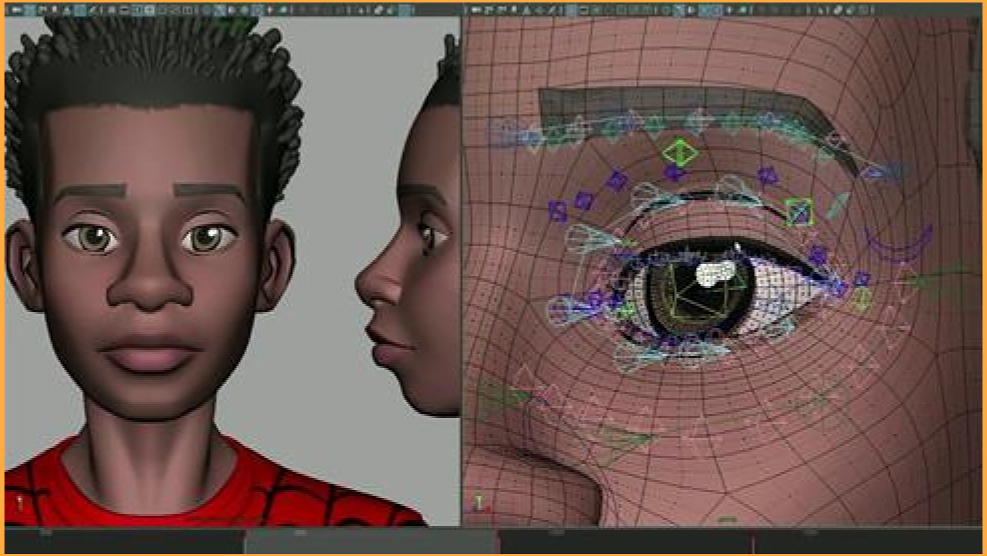
- Communicate to the computer and software how to generate and apply the motion to objects, particles, etc.

```
var initialiseArrays = function() {
    x = new Array(N);         // Positions
    y = new Array(N);
    vx = new Array(N);        // Velocities
    vy = new Array(N);
    vhx = new Array(N);       // Half step velocities
    vhy = new Array(N);
    ax = new Array(N);        // Accelerations
    ay = new Array(N);
    rho = new Array(N);       // Densities
};

var randomInit = function() {
    for (i = N; i--;) {
        // Initialize particle positions
        x[i] = random(h, 0.25);
        y[i] = random(0.5, 0.95);

        // Initialize particle velocities
        vx[i] = random(-0.02, 0.02);
```

- Create equations to represent the environment that you are trying to create.
    - Everything in 3D environments are created by the programmer, including elements like gravity. You do not need to follow conventional laws of physics.

WHERE IS THE MATH?

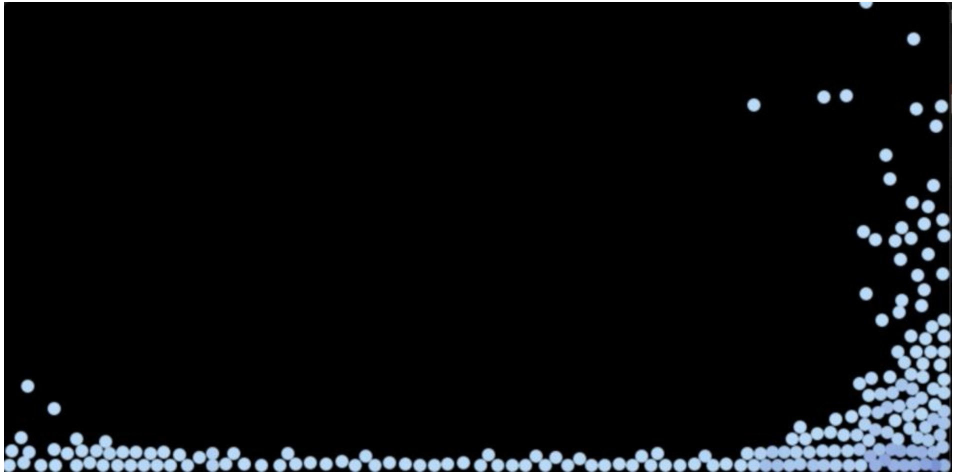
- Topologies of the models. These simulations are applied onto vertices of objects, therefore we must consider the geometric relationships between them.



- Computer is constantly computing and storing the positions and other attributes of these particles when executing simulations. Takes a lot of power.

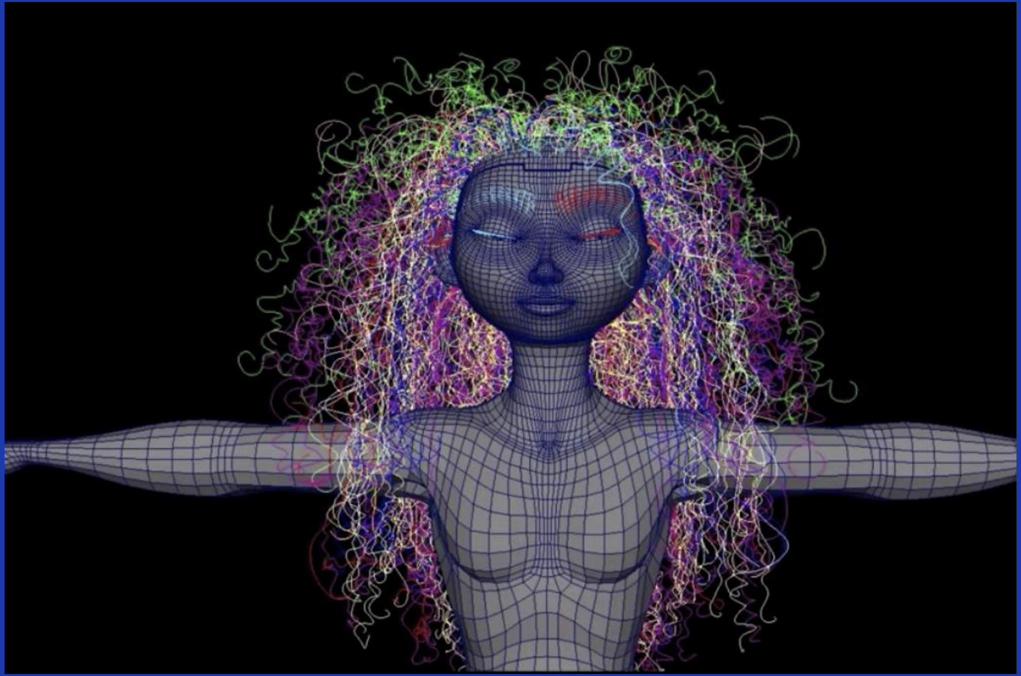
WHY DO WE USE SIMULATIONS?

- Efficiency
    - In 2D animation, we would have to hand draw each of these effects.
    - Combinatorics- helps with crowds and prevents character modelers from having to create hundreds of different characters. Simulations apply different colors, hairstyles, clothes, etc. randomly.
- Realism
    - Realistically emulate the motions of nature.
    - Can integrate with live action film.
- Exaggeration and Appeal
    - Effective for entertainment purposes. Can create cartoony effects.



https://www.khanacademy.org/computer-programming/spin-off-of-smoothed-particle-hydrodynamics/4661787965652992

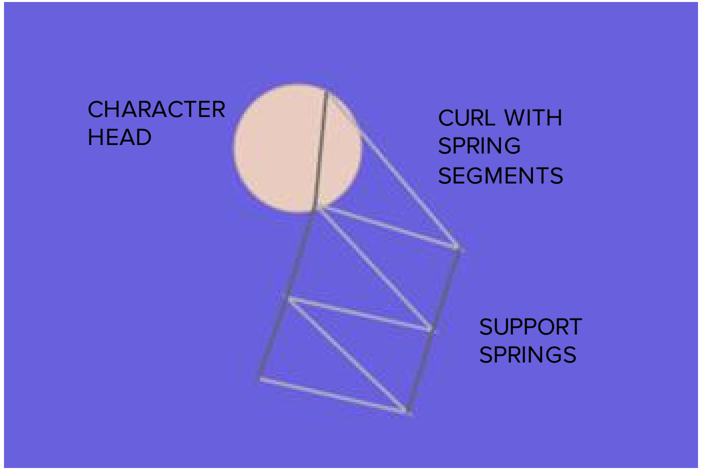
CHARACTER EFFECTS



- Different because they also have to follow the mesh of the character
- Often need to be exaggerated or have their own environment for appeal purposes

CURLY HAIR SIMPLE SIMULATION

- Break down into simple components
  - Instead of using one long hair, each piece is broken down into segments - chains of springs
  - Had to invent additional support springs to simulate what real hair does. Must be INVENTIVE!



- Spring-mass system
  - Turns out to be simple spring mass system.
  - In a real production, you would then play with the variables to get the desired look and assign attributes to make it look like hair.

https://www.khanacademy.org/computing/pixar/simulation/hair-simulation-101/a/mass-spring-system-with-support-springs

CURLY HAIR SIMPLE SIMULATION

- Hooke's Law: F = -kx
  - F= Spring force k= Spring constant x= Spring displacement (stretch or compression)
  - Find the displacement to calculate the positions of the vertices in the simulation in real time
    - Assuming the environment generates a net force on the particles (gravity), use these factors to calculate the acceleration and velocity of each particle. The net force includes the force the surrounding springs would contribute.
    - Velocity equation: v= Δx/Δt
      - This would yield the displacement of the particle from its starting position and now the computer knows where to display this object. This is too taxing for a human to calculate- hence the need for simulations

CLOTHING

- Articles of clothing are made up of vertices
- When running the simulation each vertex is responding to its corresponding equations
- Here we see:
  - Gravity
  - Relationships to other vertices (now a web rather than a string)
  - Collision with human model
  - Initially assigned position in 3D space

Selected Total
build
Plane.3
Plane.1
Plane.2
Plane
Symmetry
Plane
Body__split_4
panties
Subdivision Surface
Cloth Surface
Plane
top dynamic
top dynamic
old top
top dynamic.1
panties.1
panties.2
Brush
Mode
Edit
User Data
Cloth Tag [Cloth]
Basic
Tag
Forces
Dresser
Cache
Expert
Basic Properties
Name
Cloth
Layer
Simulation
Generators
100
Enable
5
Position
Size
Rotation
Grid Spacing : 100 cm

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Tag Properties
Auto
Start
Stop
Iterations
Stiffness
Flexion
Rubber
Bounce
Friction
Mass
Size
Tear
Use Tear
Map
Position
Size
Rotation
Grid Spacing : 100 cm
5


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build
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top dynamic.1
panties.1
panties.2
Brush
Mode Edit User Data
Cloth Tag [Cloth]
Basic Tag Forces Dresser Cache Expert
Tag Properties
Auto
Start
Stop
Iterations 15
Stiffness 100 % Map
Flexion 25 % Map
Rubber 0 % Map
Bounce 30 % Map
Friction 70 % Map
Mass 1 Map
Size 90 % Map
Tear Map
Use Tear
Position Size Rotation
5
Grid Spacing : 100 cm

SIMULATING CURTAIN WITH PARTICLES

- Things to consider:
  - Collision
  - Mass of each bead
  - Springs connecting the beads
- Things to ignore:
  - Webs or meshes of clothing
    - For the purpose of the simulation, I am returning back to individual chains rather than a web of vertices

GOALS

- Create believable simulated motion
    - In the examples previously shown, attributes have been assigned to the particles and objects for believability. I will not be creating these attributes, only attempting to apply the equations to them.
    - Attributes could be:
        - Color, glow, scale, opacity, shadows, etc.
- Interactivity + Collision
    - Not only simulate the curtain with the relationships between the particles, have the curtain react realistically when something moves through it.

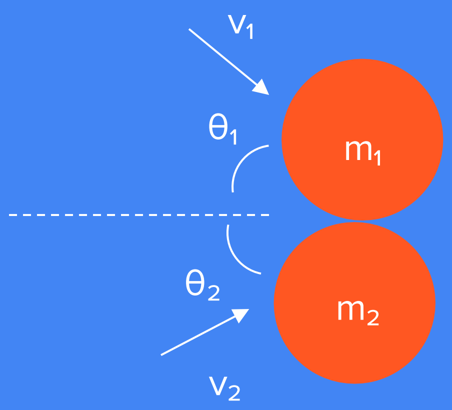
ADDITIONAL VARIABLES

- Collision
  - Newton’s Third Law of Motion
    - Upon a collision between two of these objects, the force acted on one vertex, will be matched in the opposite direction. Therefore, their motion paths will be reversed.
    - Need to take this into account when calculating their position if a bead encounters another bead AND/OR the character.

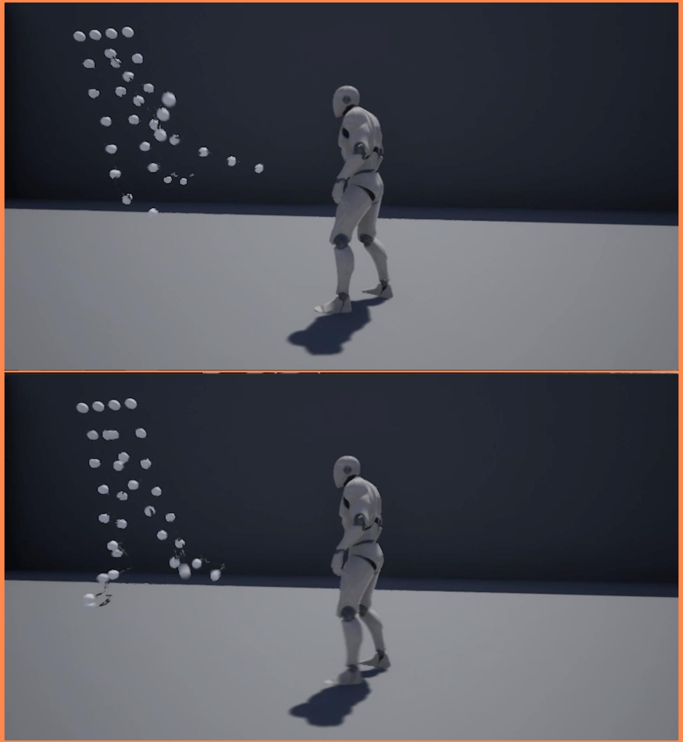
BUT these objects don’t always collide at a perfect perpendicular.

ADDITIONAL VARIABLES

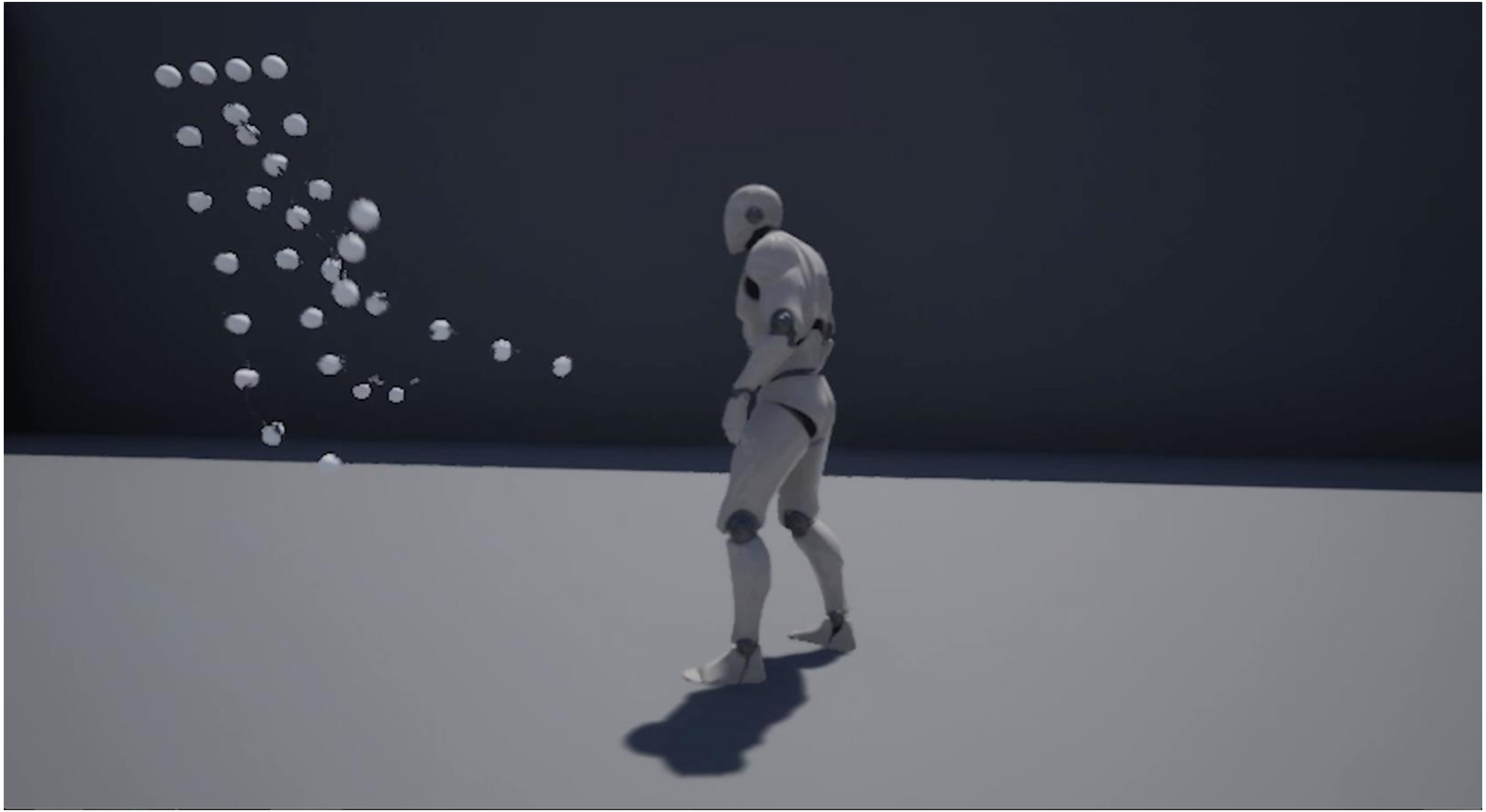
- Collision
  - $M_1v_1 = m_1v'_1 \bullet \cos(\theta_1) + m_2v'_2 \bullet \cos(\theta_2)$
    - Where:
      - m_1, m_2 = masses of the objects
      - v_1, v_2 = initial velocities
      - v'_1, v'_2 = subsequent velocities
      - θ_1, θ_2 = angles adjacent to the relative perpendicular axis
  - The program has the initial positions, mass and velocities for the objects. This is used to calculate the resulting forces angle, and velocity.

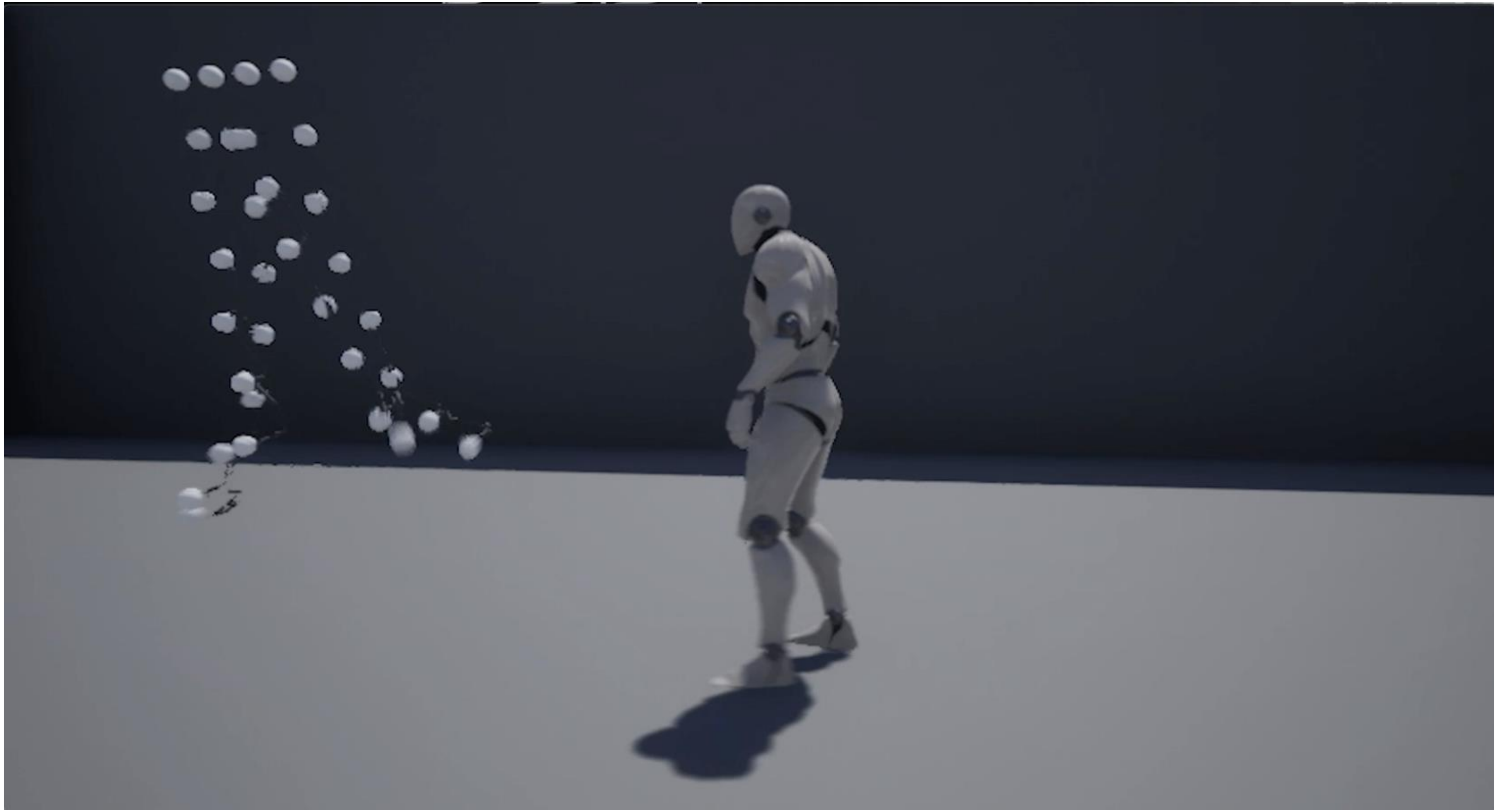


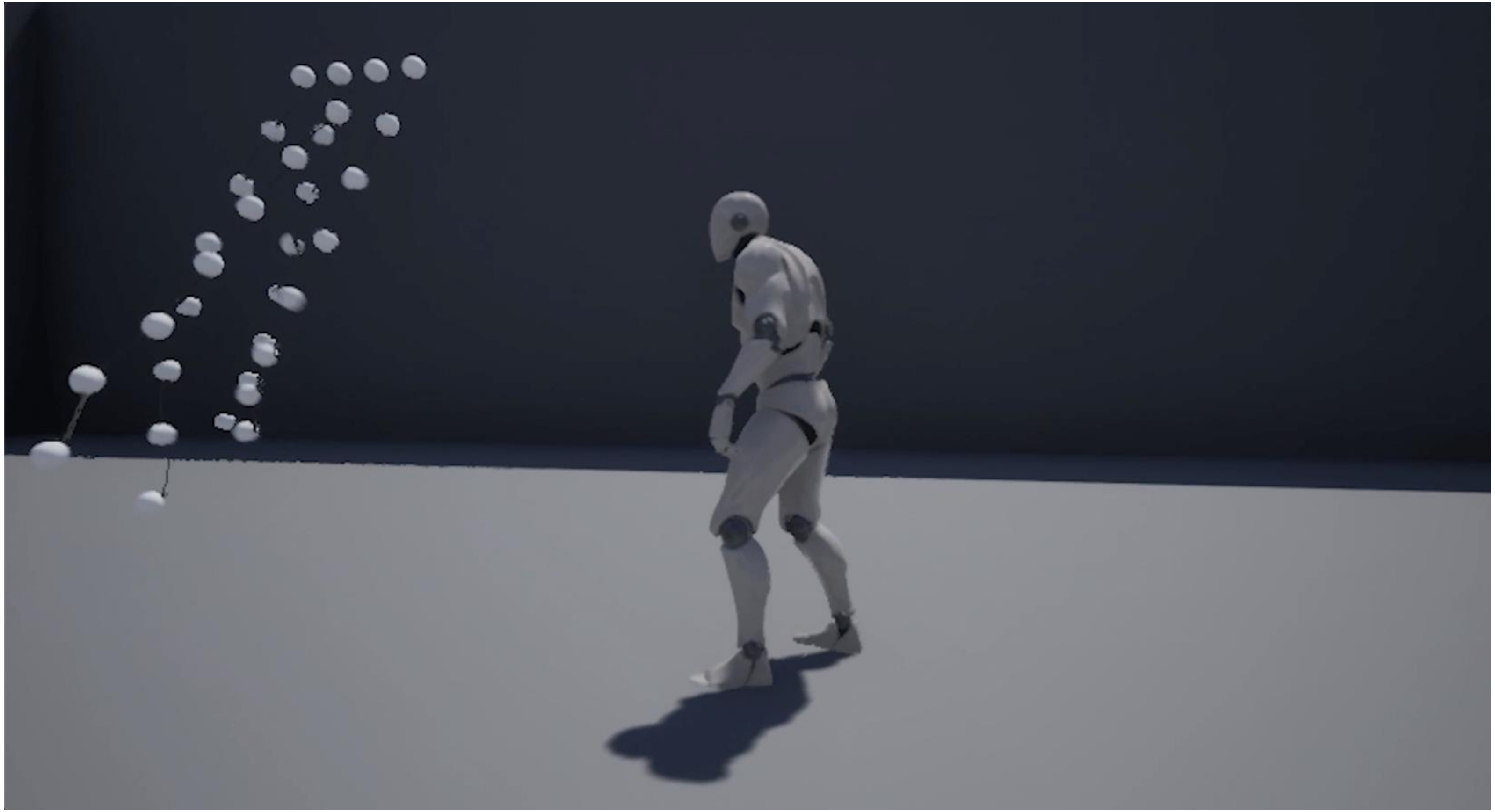
CURTAIN SIMULATION

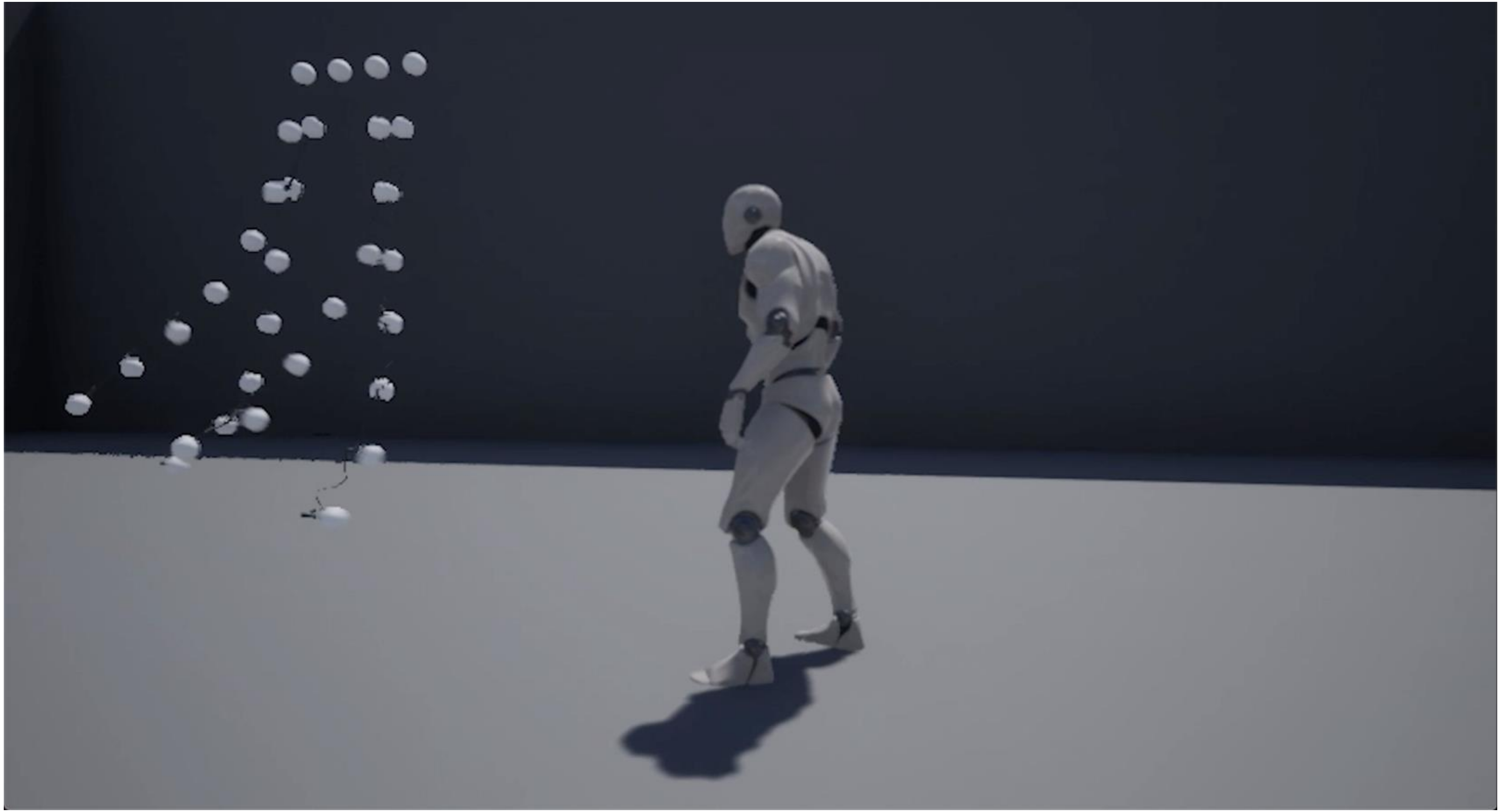


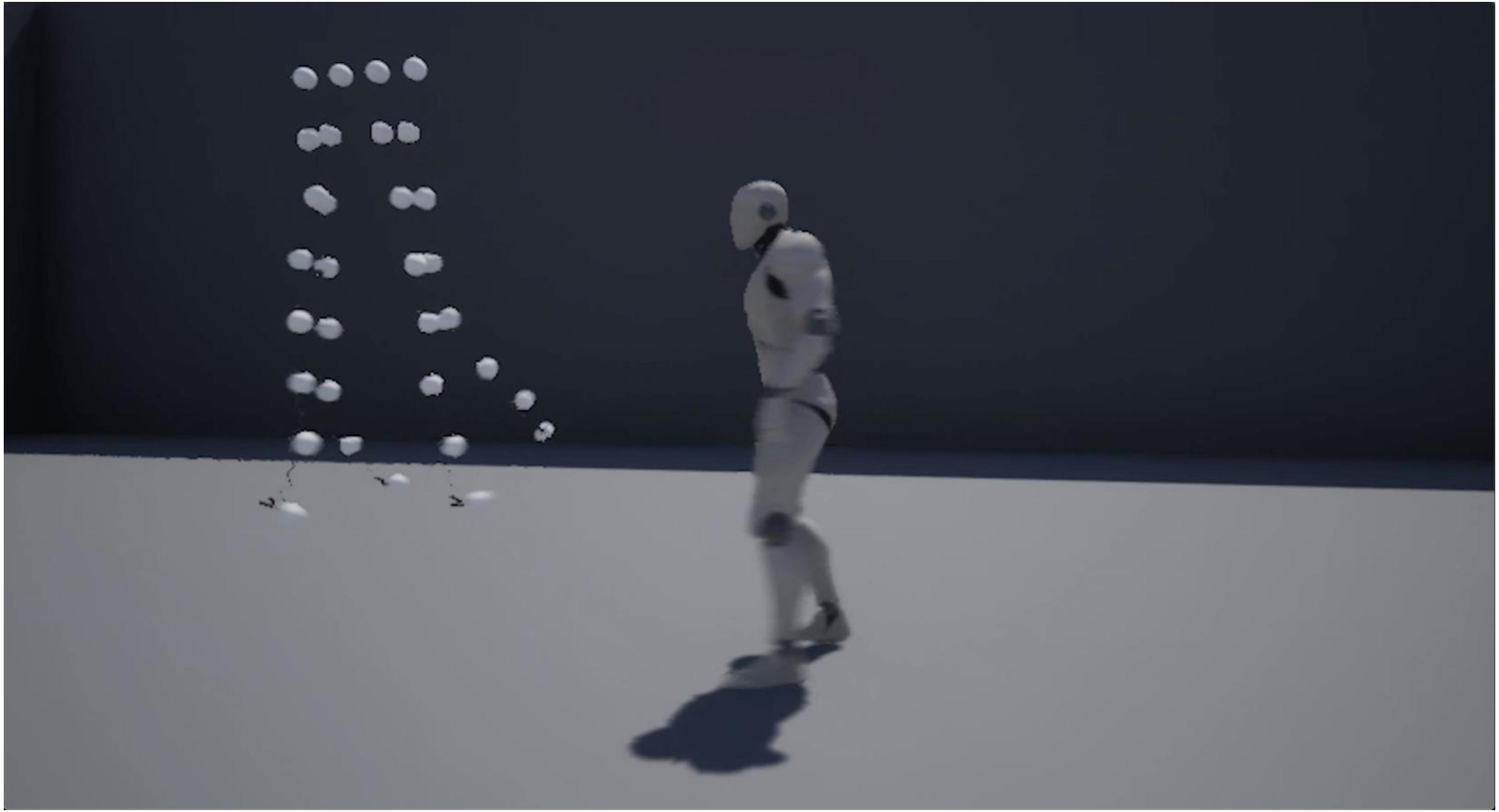
- Reacting well to each other and simulating gravity
- The bead curtain became less believable when the character encountered it. The collision was interacting with too many vertices and overwhelming the simulation.





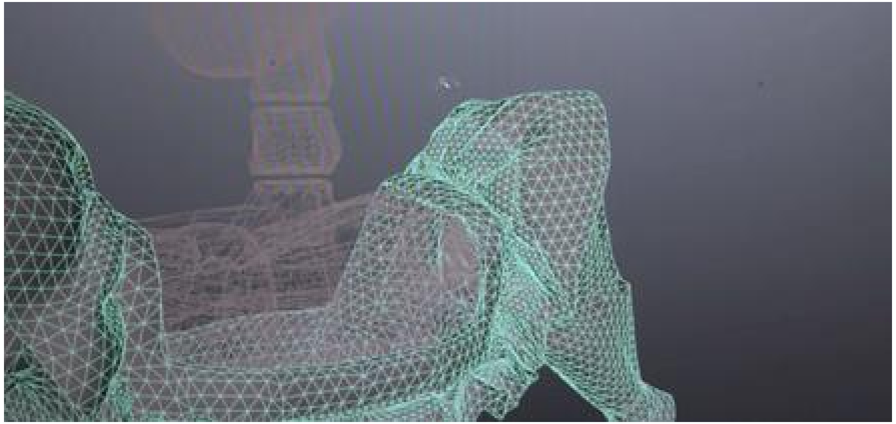






HOW TO EXPAND THIS

- Use vertices as subdivisions of planes to create a volume or web- known as a mesh or topology
- Then instead of a bead curtain, it would act as a hanging sheet of cloth - move as one



SOURCES

- https://www.khanacademy.org/computing/pixar
- https://threejs.org/examples/#webgl_animation_cloth
- https://www.britannica.com/science/Hookes-law
- https://www.britannica.com/science/Newtons-laws-of-motion